Uniformity and Efficiency of a Wireless Sensor Network’s Coverage

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Abstract—The primary contribution of this paper is in a wireless sensor network’s coverage analysis method, which focuses on both the coverage itself and its uniformity and efficiency. Sensor death is typical in the life cycle of a wireless sensor network. Hence it is important to take note of what portion of the target region is monitored by the sensor network both initially and eventually. This paper proposes a useful wireless sensor networks’ coverage analysis method, which not only focuses on the coverage itself, but also on its uniformity and efficiency. In the process, several questions such as whether the monitored region is uniformly distributed throughout the target region, or whether there are locations in the target region that are overmonitored, are answered. The paper also specifies for which type of applications the proposed coverage analysis method will be most suitable.

I. INTRODUCTION

An important application domain of wireless sensor network (WSN) is continuous monitoring. This includes application where a physical field of interest needs to be monitored for specific parameters (e.g., temperature, humidity) and must be periodically reported to a base station (BS).

A WSN that is specifically designed for continuous monitoring application, typically comprises many sensors - each capable of monitoring its vicinity for the desired parameter(s). Each such sensor sends its sensed data to the BS once every fixed/variable time interval - called a round, following a data routing path. From now on, we will refer to a WSN designed for continuous monitoring application as simply WSN.

Since a WSN is expected to be functional for an extended period of time and since sensors are typically locally powered, some of them may exhaust energy before the WSN lifetime ends. When exactly a sensor will die is determined by several factors including its initial energy and the data routing protocol used. Regardless of the specific factors, it is important to take note of what portion of the WSN is actually monitored both initially and eventually. The fraction of the WSN that is monitored by some sensor is informally referred to as coverage[1].

Figure 1 shows two scenarios of 50% WSN coverage. As can be seen, while scenario 1 gives correct information about the lower half of the target region, it completely ignores the other half. Scenario 2 on the other hand gives information about every part of the target region, though not necessarily with 100% details. The second scenario is more desirable in continuous monitoring application because it gives more accurate information about the overall status of the target region. A related problem is overmonitoring. A location in the target region is considered overmonitored if it is monitored by more sensors than required. Such coverage is simply redundant.

![Fig. 1. 50% sensing coverage.](image)

II. COMPUTING REDUNDANT AND EFFICIENT COVERAGE RATIOS

A. Coverage Ratio (C)

We define SNarea to be the physical points in the WSN that are meant to be monitored by some sensor1. So SNarea is a set of discrete points and/or continuous regions. We recommend the SNarea to be represented in a 2-dimensional (2-D) space. A real world 3-dimensional (3-D) SNarea can be mapped to such 2-D SNarea by simply projecting all points

1Here, the idea of a point is more abstract than just a point. It could correspond to a small physical volume. However, all points must be homogeneous.
on the x-y plane (which is of course a 2-D plane). In case when two distinct points in the 3-D $SN_{area}$ have different z-coordinates (i.e., different heights), they must be mapped to two different points in the 2-D space. This may force the shape of the 2-D $SN_{area}$ not to have a 1-to-1 correspondence with the shape of the actual 3-D $SN_{area}$. However, such mapping will eventually simplify other computations.

The problem with such a model is that, if in case two distinct 3-D points are monitored by the same sensor and are mapped to two 2-D points in a way that at least one of them falls outside the continuous monitoring region of and around that sensor, then the probability that any of these points is monitored by a sensor will not be a simple function of the distance between that point and a sensor. If such mapping is frequently required, then we may better off using a 3-D $SN_{area}$.

Fortunately though, the typical $SN_{area}$ for continuous monitoring application is describable by a contour map. As a result the typical shape of a 2-D $SN_{area}$ will have a 1-to-1 correspondence with the shape of its real world counterpart. So we will refer to a 2-D $SN_{area}$ as simply $SN_{area}$.

B. Possible Types of Coverage Analysis

1) Check-all-points: This is the most appropriate style of coverage analysis if the $SN_{area}$ is described as a set of only discrete points. Here, at every round of data gathering, we check whether the points in the $SN_{area}$ are monitored by some sensors.

2) Check-few-points: This style is appropriate if the $SN_{area}$ is described as a set of only continuous regions. Since any 2-D region (no matter how small it is) has infinitely many points, we have to be selective in checking regions.

3) Mixed-point-selection: Suitable for $SN_{area}$ that contains both discrete points and continuous regions. Here coverage analysis is a combination of the last two approach. That is, we check all discrete points in addition to the few randomly selected points from the continuous regions.

Let $SN_{area} = \{X_1, X_2, \ldots, X_p\} \cup \{X_{p+1}, X_{p+2}, \ldots, X_{p+r}\}$, where $X_i$ is a point if $1 \leq i \leq p$ and a region otherwise. Assume we are using Mixed-point-selection strategy. So our analysis will be a check-all-points analysis if $r = 0$, and a check-few-points analysis if $p = 0$.

The point selection process is actually a two-step procedure. First, we check all $p$ points: $X_1$ to $X_p$ whether they are monitored by some sensor. Then, we do either of the followings:

1) We select $n$ random points from $X_{p+1}$ to $X_{p+r}$ (similar to Monte Carlo algorithm [2]). $n$ can be adjusted based on the total area of all $r$ regions. The larger the value of $n$, the better the coverage analysis is.

2) We select $n_k$ random points from each region $X_k$, where $n_k$ is proportional to the area of $X_k$.

The last option makes more sense since otherwise we may end up selecting more points from the smaller regions.

However, the last option has its own drawback too. To see how, consider two WSNs: $SN_1$ and $SN_2$ (having $SN_{area}$s: $SN_{area1}$ and $SN_{area2}$, respectively). Let the total area of the regions in $SN_{area1}$ be equal or less than the area of some region in $SN_{area2}$. Given this condition, if we use the first alternative for $SN_1$, we would have the same problem in both $SN$s even if we use the second approach for $SN_2$ (because $SN_{area2}$ contains a region with area greater than or equal to the total area of all the regions in $SN_{area1}$).

As the sensors die, a good coverage analysis must also reflect sensors’ death pattern (i.e., centralized or dispersed). This is necessary because sensor nodes may die mostly in small but specific parts of the $SN_{area}$. If we randomly check points from a large region, no matter how many points we choose, we may still have these specific small death zones unchecked. Albeit, in this way we may find great sensing coverage while some parts of the $SN_{area}$ is completely unmonitored! The situation may seem odd but it’s not unlikely.

1) A More Appropriate Point Selection Strategy: Let the definition of $SN_{area}$ be as we have already mentioned. We will check all $p$ points: $X_1$ to $X_p$. And additionally we will check maximum $n$ points from all $r$ regions: $X_{p+1}$ to $X_{p+r}$.

We begin by embedding the $SN_{area}$ (excluding the discrete points) into a square (see Figure 2). Let $s$ be the length of this square’s each side. The square need not be minimum. That is, $s$ need not have an upper bound; although to avoid unnecessary computation, we would expect the square to be only large enough to contain $SN_{area}$’s regions. The upperbound of $n$ (i.e., the number of points to check) should be determined only by the size of this square, not by the size of regions in $SN_{area}$. However, to get an accurate analysis, $n$ must be proportional to the size of the embedding square.

![Fig. 2. Embedding $SN_{area}$ (excluding the discrete points) into a square.](image)
Now, divide the embedding square into $m^2$ smaller equal squares. Each of these squares should have sides of length $\frac{\sqrt{m}}{m}$, and therefore their common area will be $\left(\frac{\sqrt{m}}{m}\right)^2$. We index these small squares as $M_{f,g}$, where $1 \leq f \leq m$ and $1 \leq g \leq m$.

Define $F_{f,g}$ to be the fraction of $M_{f,g}$ that is part of $SN_{area}$. That is,

$$F_{f,g} = \frac{\text{Total area of } SN_{area} \text{ that is within } M_{f,g}}{\left(\frac{\sqrt{m}}{m}\right)^2} \quad (1)$$

Now let

$$SqrInSN = \frac{\sum_{i=(p+r)}^{(p+r)} \text{area}(X_i)}{S^2} \quad (2)$$

$SqrInSN$ represents the fraction of the embedding square that is within $SN_{area}$. Given all these information, we define Coverage Ratio($C$) as follows:

$$C = \sum_{i=1}^{p} \text{CovProb}(X_i, \text{MaxAccuracy})$$

$$+ \sum_{j=1}^{m} \sum_{g=1}^{m} \sum_{h=1}^{F_{f,g} \times n} \text{CovProb}(A\text{UniqueRandomPoin}t\text{In}(M_{f,g}), \text{MaxAccuracy}))$$

$$/ (p + (n \times SqrInSN)) \quad (3)$$

$C$ tells us what portion of the $SN_{area}$ is monitored by some sensors. The CovProb function (Figure 3) computes the probability of a point being monitored by some sensor. The function uses $\text{MaxAccuracy}$ to determine the monitoring probability of a point. For example, if $\text{MaxAccuracy} = 5$, the function considers a point fully monitored only if it is monitored by at least 5 sensors (or with equivalent total monitoring probability by more than 5 sensors). The function does not give extra credit for extra monitoring (e.g., when a point is perfectly monitored by more than 5 sensors).

Function CovProb(p2d: a point in the 2-D $SN_{area}$, max_prob: MaxAccuracy)

$p3d := \text{map}(p2d \text{ to the real world point})$

$\text{total}_\text{prob} := 0$

for each sensor $s$

$$\text{total}_\text{prob} := \text{total}_\text{prob} + \text{probability}(s \text{ can monitor } p3d \text{ given distance}_\text{between}(s, p3d))$$

return minimum$(1, \text{total}_\text{prob}/\text{max}_\text{prob})$

Fig. 3. The CovProb function: It returns the probability of a point being monitored by some sensor. Notice that the function takes a 2-D point from the $SN_{area}$ but maps it to the actual 3-D point before computing the distance between that point and a sensor.

C. Redundant Coverage Ratio (RC)

We define Redundant Coverage Ratio ($RC$) to be the fraction of $SN_{area}$ that is overmonitored (i.e., monitored by more than MaxAccuracy). The equation to compute $RC$ is given below. The IsRMon method is a modified version of the CovProb function. It returns 1 when total_prob is greater than 1 (see figure 3). Otherwise, it returns 0.

$$RC = \sum_{i=1}^{p} \text{IsRMon}(X_i, \text{MaxAccuracy})$$

$$+ \sum_{j=1}^{m} \sum_{g=1}^{m} \sum_{h=1}^{F_{f,g} \times n} \text{IsRMon}(A\text{UniqueRandomPoin}t\text{In}(M_{f,g}), \text{MaxAccuracy}))$$

$$/ (p + (n \times SqrInSN)) \quad (4)$$

Remember though a positive value of $RC$ does not imply that some portion of $C$ is completely valueless. For example, $C = 0.8$ and $RC = 0.2$ should not be interpreted as $C - RC = 0.8 - 0.2 = 0.6$ or 60% is the actual coverage. Rather, the appropriate interpretation would be that 80% of $SN_{area}$ is monitored and 20% of the $SN_{area}$ is monitored by more sensors than necessary.

D. Efficient Coverage Ratio (EC)

Efficient Coverage Ratio (EC) is the difference between $C$ and $RC$. That is,

$$EC = C - RC$$

EC tells us what portion of the $SN_{area}$ is monitored with exactly MaxAccuracy. Clearly, this is the number that any WSN sensor distribution or data routing algorithm should try to maximize.

E. Getting Good Initial Coverage

The $C$ of $N$ randomly distributed sensor nodes can be approximated when their common sensory range $r$ is known\(^2\). Let’s assume a point be monitored or covered if it is monitored by at least one sensor. Then, the following equation can be used to get a lower bound of $N$ for an expected initial $C$ [3].

$$\begin{align*}
\text{Expected}(C) = & \left(1 - \left(1 - \frac{0.5r^4 - 1.333Lr^3 - 1.333W^3r^3 + 3.14LW^2r^2}{L^2W^2}\right)^N\right) \\
\end{align*} \quad (5)$$

Here $L$ and $W$ are the length and width of the $SN_{area}$, respectively. However, such parameters only exist if $SN_{area} = \{\text{A rectangular region}\}$. Further, this equation only works

\(^2\)Sensory range is the maximum distance from a sensor within which all points are perfectly monitored by that sensor.
with sensors that are uniformly distributed at random. Fortunately, in continuous monitoring applications where large number of sensors are typically deployed, random distribution is particularly useful. This is because as the number of sensors increases, the benefit difference between careful and random sensor deployment decreases too.

Finally, equation 5 does not ensure that no point in the $SN_{area}$ will be monitored by more than one sensor. Put differently, the same initial $C$ could be achieved with a smaller value of $N$ using a careful sensor distribution.

Figure 4 shows a simple algorithm that uses this equation to give an approximate lower bound of the number of sensor nodes needed to get an expected initial coverage. The algorithm performs more efficiently with a range of expected $C$ than an exact value.

Function func (max. $C$; Max Expected Initial Coverage Ratio, min. $C$; Min Expected Initial Coverage Ratio increment, decrement)

$N := 0$
$curre_C := 0$

while(true)

if $curre_C < min. C$ then

$N := N + increment$

else if $curre_C > max. C$ then

$N := N - decrement$

else

break

curre_C := get Expected(C) putting $N$ in equation (5)

end while

return $N$

Fig. 4. A function that computes the minimum number of sensors required to get an expected initial coverage ratio when distributed randomly.

III. EXPERIMENTAL RESULTS: PLOTTING COVERAGE RATIOS AND INTERPRETING THEIR CHANGE

An energy efficient data routing protocol is proposed primarily for continuous monitoring WSNs in [4]. In this section, we will use the coverage analysis method of this paper to compute the coverage ratios of a WSN that uses the data routing protocol of [4]. We will also demonstrate how to plot and interpret the results.

Figure 5 to 8 show the simulation results. The simulation setup was similar to the one used in [4]. The exact simulation setup is indeed not important here. What is important is how we analyze the obtained results: Active to total sensors ratio (ATS), coverage ratio (C), and redundant coverage ratio (RC) against the rounds of WSN’s data gathering. In the experiments, we used two different scenarios with same data routing protocol but with different number of sensors and random sensor deployments. Figure 5 and 6 correspond to the first scenario ($SCN_1$). We will refer to figure 5, 6, 7, and 8 as $SCN_1 − 1$, $SCN_1 − 2$, $SCN_2 − 1$, and $SCN_2 − 2$ respectively.

A. Comparative Analysis

1) $SCN_1 − 1$ and $SCN_2 − 1$ plot ATS, C, and RC against the WSN’s round of data gathering. This type of plotting are appropriate when the goal is to evaluate the uniform coverage of a WSN and the redundancy in it. On the other hand, $SCN_1 − 2$ and $SCN_2 − 2$, which plot ATS and EC against round, are more appropriate for solely evaluating WSN’s coverage efficiency.

2) In our experiments, $MaxAccuracy$ was 1. This means that we considered a point in the target region (or more precisely, in the $SN_{area}$) monitored if it is monitored by at least 1 sensor, and overmonitored if it is monitored by more than 1 sensor. As can be seen in both $SCN_1 − 1$ and $SCN_2 − 1$, there were some initial redundancy in the WSN’s coverage in both scenarios, which means that
some points in the target region were initially monitored by more than 1 sensor. This is reasonable since sensors were deployed uniformly in random.

3) The decrease rates of both $C$ and $RC$ in $SCN1 - 1$ and $SCN2 - 1$ can be analyzed as follows:

- When $RC$’s decrease rate is negligible compared to $C$’s one (as is the case in the early rounds of $SCN1 - 1$), then it can be inferred that the initial coverage redundancy was intense and localized. To see why, assume the overmonitored locations in the target region are monitored by at least $sr$ sensors. Now if $C$ decreases, $RC$ will remain constant until all redundant sensors in at least one overmonitored location die. Since $sr$ can be $>> 1$, even if the data routing protocol is smart enough to kill the redundant sensors early by overusing them to route other sensors’ data, $RC$ will remain constant for substantial number of rounds. Such pattern is indeed unexpected since the redundant coverage is desirable to be minimized as early as possible.
- The second important pattern is revealed when $RC$’s decrease rate equals $C$’s one. This is in fact the best possible case in our experiments since $MaxAccuracy$ was 1 (i.e., $RC$ could not decrease faster than $C$).
- The intermediate case occurs when both $C$ and $RC$ are decreases at a different rate (as in $SCN2 - 1$).

4) Both $C$ and $RC$ have different initial values in $SCN1 - 1$ and $SCN2 - 1$. Further, their corresponding graphs in the two figures are quite different. Despite this fact, the graphs of $EC$ in $SCN1 - 2$ and $SCN2 - 2$ are almost identical. There are two important implication of this phenomenon:

- The WSN’s data routing protocol gives consistent coverage efficiency change when sensors are randomly deployment initially.
- The sensor deployment strategy gives an upper-bound of initial coverage efficiency. That is, to get more coverage than this upperbound, some redundant coverage must be introduced. The remedy is to use careful (not random) sensor deployment that increases initial coverage without increasing redundancy.

IV. CONCLUSION AND FUTURE WORK

This paper proposes a WSNs’ coverage analysis method. To summarize, it defines three terms that have the following interpretations: a coverage ratio of 0.7 means 70% is monitored by the WSN, and this monitored region is uniformly distributed in the target region; a redundant coverage ratio of 0.2 is another way of saying that 20% of the target region is monitored by more sensors than necessary; an efficient coverage of 0.5 implies that 50% of the target region is monitored exactly by as much sensors as necessary.

In future, we would like to investigate the sensing coverage of mobile sensor networks.

REFERENCES